Forcing Causality on S-Parameter Data Using the Hilbert Transform

Philip A. Perry, *Member, IEEE,* and Thomas J. Brazil, *Member, IEEE*

***Abstract*— A new technique to condition S-parameter data for inclusion in a time-domain simulator is presented. This technique uses the Hilbert transform and a new set of network parameters which are always positive real for a passive network. The usefulness of the technique is demonstrated by removing the noncausal aspects of a set of measured S-parameter data.**

***Index Terms*— CAD, Hilbert transform, S parameters, time-domain simulations.**

I. Introduction

RECENT interest in the time-domain simulation of nonlinear microwave circuits and systems has been largely motivated by the increase in clock speed of digital circuits. The need to simulate the distortion of digital signals as they propagate along transmission lines has presented perhaps the single greatest challenge in this field, as such structures are usually modeled in the frequency domain using semi empirical models.

The inclusion of devices described by frequency-domain data in the form of measured or simulated S-parameter data is usually achieved through a convolution-based technique [1]. Thus, the frequency-domain data must be transformed to a time-domain impulse response representation using an algorithm which is usually based on the fast Fourier transform (FFT). The generation of the impulse response is usually an integral part of the convolver, several of which are now commercially available, all of which require a high level of user competence.

II. Passivity and Causality

If the problem is viewed as that of including S-parameter data blocks in a time-domain simulator in this way, then it is clear that the circularity of the FFT algorithm will force the frequency-domain data and the time-domain data to be periodically extended. This can cause difficulties through the presence of discontinuities at the end of the frequency domain data. Windowing is often used to avert these problems.

Such S-parameter data may contain errors due to the use of a model beyond its recommended range of frequencies or, in the case of measured data, due to the presence of noise or systemic errors. In either case, these errors can lead to a noncausal impulse response. The noncausal part of the response must be removed by further windowing if serious aliassing errors are to be avoided [2]. Such windowing in the time domain means that the resulting impulse response is not the impulse response of the original frequency domain data. Moreover, the original data is rarely truly periodic, so that it *must* be modified in some way to achieve a causal impulse response.

The goal of this work was to modify the frequency-domain data so that windowing in the time domain was not necessary. This can be achieved through the use of the Hilbert transform to calculate the imaginary part of each S parameter from a knowledge of its real part or vice versa. This approach can lead to a modification of the magnitude of the frequency domain data which can become greater than unity. A passive circuit with an S parameter greater than one is obviously nonphysical.

Another form of the Hilbert transform relates the phase of a positive real (*p.r.*) function to its magnitude. This has been used by the authors to calculate a relative group delay for networks with minimum phase transfer functions [3]. Reflection S parameters, however, are rarely *p.r.* so that this form of the Hilbert transform cannot be directly applied to an S-parameter block. By applying an offset of one to each S-parameter value, however, all elements in the S matrix of a passive network can be modified to produce a *p.r.* function. In this case the authors have used "$1 - S$" parameters although "$1 + S$" or "$S - 1$" would also be *p.r.* functions.

Using the "$1 - S$" parameters it is possible to calculate their phase from their magnitude data using the Hilbert transform so that they represent causal functions. Using an FFT-based version of the Hilbert transform [3] the function is also forced to be circular. Since the S parameters of a passive network are bounded to plus or minus one, the "$1 - S$" parameters are bounded between zero and two. Thus, it is possible to bound the parameters to ensure passivity is maintained or enforced.

III. Results

An algorithm has been developed which reads an S-parameter file, generates the "$1 - S$" parameters and calculates their phase, then converts the result back to S parameters for inclusion in a commercial time domain simulator. These modified S parameters have been termed the "causal S parameters." This has been tested for several networks and

Manuscript received June 8, 1998.

P. A. Perry is with the School of Electronic Engineering, Dublin City University, Dublin 9, Ireland.

T. J. Brazil is with the Department of Electronic and Electrical Engineering, University College Dublin, Dublin 4, Ireland.

Publisher Item Identifier S 1051-8207(98)09532-4.

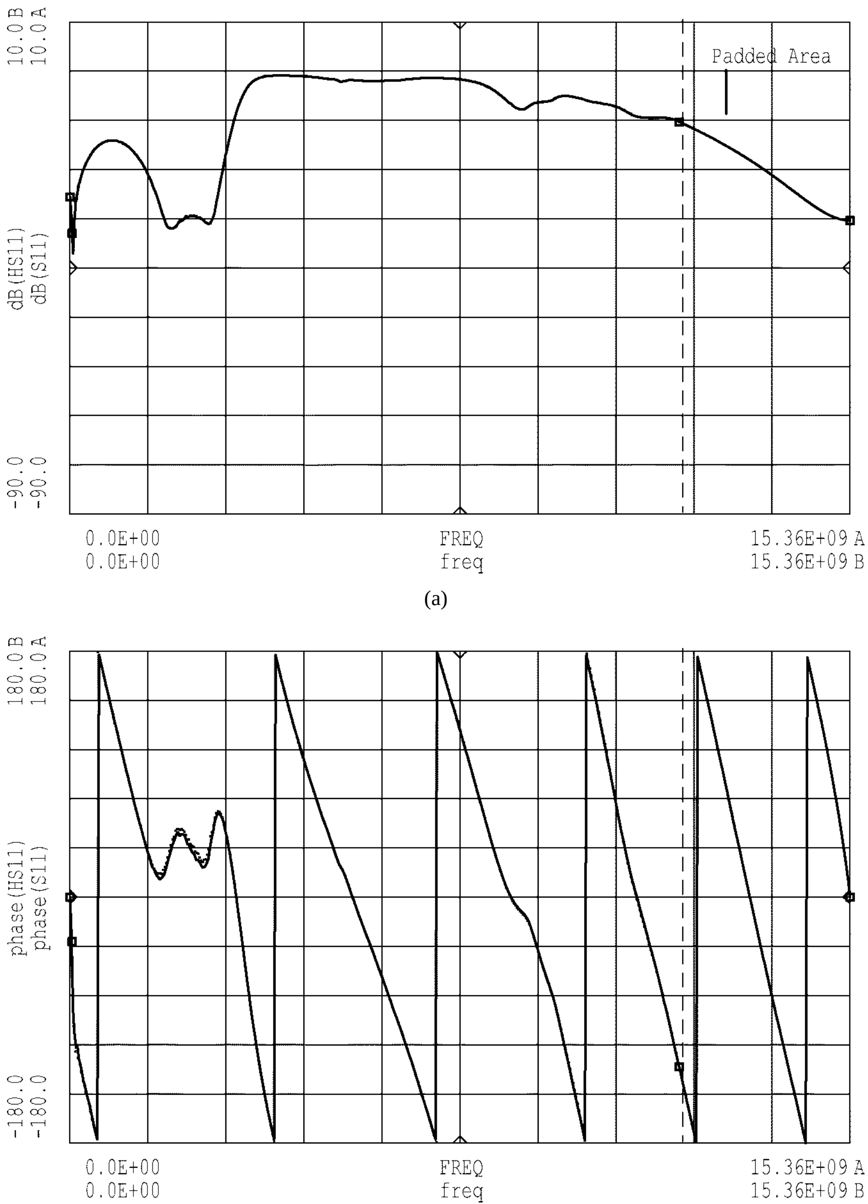

Fig. 1. (a) Magnitude and (b) phase of the measured S_{11} for the band stop filter. Raw S-parameter data (dots) and causal S parameters (solid lines).

has been found to work well. To illustrate its efficacy, the results of a measurement of the S_{11} of a band stop filter are presented. The frequency-domain data for the modified and raw data are shown in Fig. 1. It is clear that the phase of the modified data is smoother in the region where the magnitude is of the order of -30 dB. The noise on the measured data has effectively been removed.

The padded area above 12 GHz shows the additional data points which have been added to allow the magnitude to smoothly meet its periodically extended self. These extra points were added to the magnitude of the "$1 - S$" data using quartic interpolation between the data and its extended self. This area also allows the phase to smoothly change so that the final data point is purely real so there is no discontinuity here either.

The impulse responses in Fig. 2 shows the difference between that of the raw data and the modified data. It should be noted that the amplitude requires scaling prior to use in a convolution simulator. The new impulse response is now almost entirely causal and the ringing caused by the discontinuities has been removed. These responses were obtained using the Chirp-Z transform with no windowing applied so that the improvement is solely due to the application of the Hilbert transform algorithm. This new impulse response is therefore, directly related to the modified frequency domain data. Similar improvements have been observed for simulations of distributed circuits.

Using this technique, all data modification has been confined to one area and may be conveniently controlled to ensure that the results remain physically realizable. Although the main

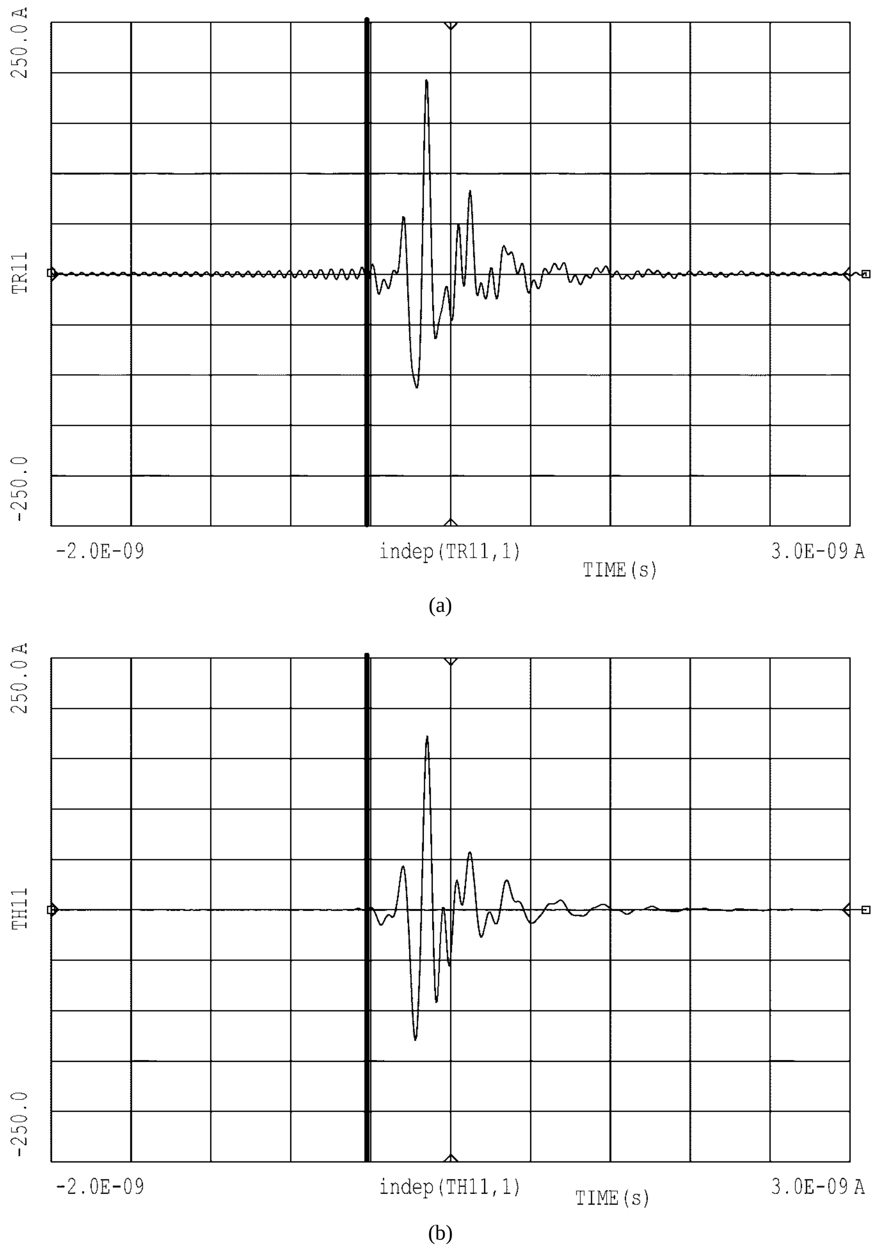

Fig. 2. (a) Impulse response from measured S_{11} for harmonic reject filter and (b) the corresponding impulse response from the derived causal S parameters.

application of the technique is in the simulation context, it appears that this technique may prove useful in many other applications such as radar and channel sounding applications.

IV. Conclusions

A new approach to the generation of impulse response data from S parameters has been proposed. This enables the use of frequency domain data in a SPICE-type time-domain simulator. This takes an holistic approach to the problem and is based on the magnitude and phase form of the Hilbert transform. Such an approach can only be applied to positive real functions which precludes its general application in the case of S parameters. Through a simple transformation to "$1-S$" parameters, a positive real function is guaranteed for all lossy distributed circuits. Results have been presented which indicate the benefits of this approach through the reduction of non causal responses and the removal of ringing effects.

References

[1] T. J. Brazil "Causal convolution—A new method for the transient analysis of linear systems at microwave frequencies," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 315–323, Feb. 1995.

[2] A. V. Oppenheim and R. W. Schafer, *Discrete-Time Signal Processing.* Englewood Cliffs, NJ: Prentice-Hall, 1989.

[3] P. A. Perry and T. J. Brazil, "Hilbert-transform-derived relative group delay," *IEEE Trans. Microwave Theory Tech.*, vol. 45, pp. 1214–1225, Aug. 1997.